DSC 40A

Theoretical Foundations of Data Science I

Today

- More examples of using combinatorics to solve probability questions.

Question

Answer at q.dsc40a.com

Remember, you can always ask questions at q.dsc40a.com!

If the direct link doesn't work, click the "Lecture Questions" link in the top right corner of dsc40a.com.

Counting as a Tool for Probability

Example 8. What is the probability that a randomly generated bitstring of length 10 is the string 0011001101?

$$P(n,k) = (n)(n-1)\ldots(n-k+1) = \frac{n!}{(n-k)!}$$

$$C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Counting as a Tool for Probability

Example 7. What is the probability that a randomly generated bitstring of length 10 contains an equal number of zeros and ones?

$$P(n, k) = (n)(n-1)\ldots(n-k+1) = \frac{n!}{(n-k)!}$$

$$C(n, k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Counting as a Tool for Probability

Example 10. What is the probability that a fair coin flipped 10 times turns up HHTTHHTTHT?

$$P(n,k) = (n)(n-1)\ldots(n-k+1) = \frac{n!}{(n-k)!}$$

$$C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Counting as a Tool for Probability

Example 9. What is the probability that a fair coin flipped 10 times turns up an equal number of heads and tails?

$$P(n,k) = (n)(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!}$$

$$C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Counting as a Tool for Probability

Example 12. What is the probability that a biased coin with $Prob(H) = \frac{1}{3}$ flipped 10 times turns up HHTTHHTTHT?

$$P(n,k) = (n)(n-1)\ldots(n-k+1) = \frac{n!}{(n-k)!}$$

$$C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Counting as a Tool for Probability

Example 11. What is the probability that a biased coin with $Prob(H) = \frac{1}{3}$ flipped 10 times turns up an equal number of heads and tails?

$$P(n,k) = (n)(n-1)\ldots(n-k+1) = \frac{n!}{(n-k)!}$$

$$C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

The Easy Way

Example 6. There are 20 students in a class. A computer program selects a random sample of students by drawing 5 students at random **without replacement**. What is the chance that a particular student is among the 5 selected students?

from Theory Meets Data by Ani Adhikari, Chapter 4

The Easy Way

Example 6. There are 20 students in a class. A computer program selects a random sample of students by drawing 5 students at random **without replacement**. What is the chance that a particular student is among the 5 selected students?

Another way to think of sampling without replacement:

1. randomly shuffle all 20 students
2. take the first 5

from Theory Meets Data by Ani Adhikari, Chapter 4

Practice Problems

Example 13. You were one of 238 individuals who reported for jury duty. If 54 of these people will be assigned to a courtroom, what is the probability that you get assigned to a courtroom?

Practice Problems

Example 13. You were one of 238 individuals who reported for jury duty. If 54 of these people will be assigned to a courtroom, what is the probability that you get assigned to a courtroom?

How many sets of 54 individuals include you?

A. C(238, 54)

B. C(237, 54)

C. C(238, 53)

D. C(237, 53)

Practice Problems

Example 14. You were one of 238 individuals who reported for jury duty. Suppose 28 of these individuals are doctors. If 54 of these people will be assigned to a courtroom, what is the probability that exactly 5 doctors get assigned to a courtroom?

Practice Problems

Example 15. What is the probability that your five-card poker hand is a straight?

Practice Problems

Example 16. Suppose you look at your first card as it is dealt, and you see that it is a Queen. What is the probability that your five-card hand is a straight?

Summary

- Counting is a useful tool for probability.
- Sometimes there’s an easy way!
- **Next time:** Bayes Theorem