DSC 40A Theoretical Foundations of Data Science I

Today

 More examples of using combinatorics to solve probability questions.



# Remember, you can always ask questions at <u>q.dsc40a.com</u>!

If the direct link doesn't work, click the "Lecture Questions" link in the top right corner of <u>dsc40a.com</u>.

**Example 8.** What is the probability that a randomly generated bitstring of length 10 is the string 0011001101?

$$P(n,k) = (n)(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!} \qquad C(n,k) =$$

$$C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

**Example 7.** What is the probability that a randomly generated bitstring of length 10 contains an equal number of zeros and ones?

$$P(n,k) = (n)(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!} \qquad C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

/ \

.

**Example 10.** What is the probability that a fair coin flipped 10 times turns up HHTTHHTTHT?

$$P(n,k) = (n)(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!} \qquad C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

**Example 9.** What is the probability that a fair coin flipped 10 times turns up an equal number of heads and tails?

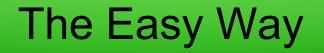
$$P(n,k) = (n)(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!} \qquad C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

**Example 12.** What is the probability that a biased coin with  $Prob(H) = \frac{1}{3}$  flipped 10 times turns up HHTTHHTTHT?

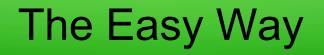
$$P(n,k) = (n)(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!} \qquad C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

**Example 11.** What is the probability that a biased coin with  $Prob(H) = \frac{1}{3}$  flipped 10 times turns up an equal number of heads and tails?

$$P(n,k) = (n)(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!} \qquad C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$



**Example 6.** There are 20 students in a class. A computer program selects a random sample of students by drawing 5 students at random **without replacement**. What is the chance that a particular student is among the 5 selected students?



**Example 6.** There are 20 students in a class. A computer program selects a random sample of students by drawing 5 students at random **without replacement**. What is the chance that a particular student is among the 5 selected students?

#### Another way to think of sampling without replacement:

- 1. randomly shuffle all 20 students
- 2. take the first 5

**Example 13.** You were one of 238 individuals who reported for jury duty. If 54 of these people will be assigned to a courtroom, what is the probability that you get assigned to a courtroom?

**Example 13.** You were one of 238 individuals who reported for jury duty. If 54 of these people will be assigned to a courtroom, what is the probability that you get assigned to a courtroom?

How many sets of 54 individuals include you?	
A. C(238, 54)	C. C(238, 53)
B. C(237, 54)	D. C(237, 53)

**Example 14.** You were one of 238 individuals who reported for jury duty. Suppose 28 of these individuals are doctors. If 54 of these people will be assigned to a courtroom, what is the probability that exactly 5 doctors get assigned to a courtroom?

**Example 15.** What is the probability that your five-card poker hand is a straight?

**Example 16.** Suppose you look at your first card as it is dealt, and you see that it is a Queen. What is the probability that your five-card hand is a straight?

## Summary

- Counting is a useful tool for probability.
- Sometimes there's an easy way!
- Next time: Bayes Theorem