DSC 40A

Theoretical Foundations of Data Science I

In This Video

The optimal prediction h^* that minimizes $R(h)$ is a measure of center. What is the meaning of the value of $R(h^*)$?

Recommended Reading

Course Notes: Supplement 1

General Approach

- We start with a loss function $L(h, y)$.
- Then we minimize the empirical risk:

$$R(h) = \frac{1}{n} \sum_{i=1}^{n} L(h, y_i).$$

- The input h^* that minimizes $R(h)$ is some measure of the **center** of the data set.
- The minimum output $R(h^*)$ represents some measure of the **spread**, or variation, in the data set.

Absolute Loss

- The empirical risk for the absolute loss is

$$R(h) = \frac{1}{n}\sum_{i=1}^{n} |y_i - h|.$$

- $R(h)$ is minimized at $h^* = \text{median}(y_1, y_2, \ldots, y_n)$.

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- $R(h)$ is minimized at $h^* = \text{median}(y_1, y_2, \ldots, y_n)$.

- Therefore, the minimum value of $R(h)$ is

$$\begin{aligned} R(h^*) &= R(\text{median}(y_1, y_2, \ldots, y_n)) \\ &= \frac{1}{n}\sum_{i=1}^{n} |y_i - \text{median}(y_1, y_2, \ldots, y_n)|. \end{aligned}$$

Mean Absolute Deviation from the Median

- The minimum value of $R(h)$ is the **mean absolute deviation from the median**.

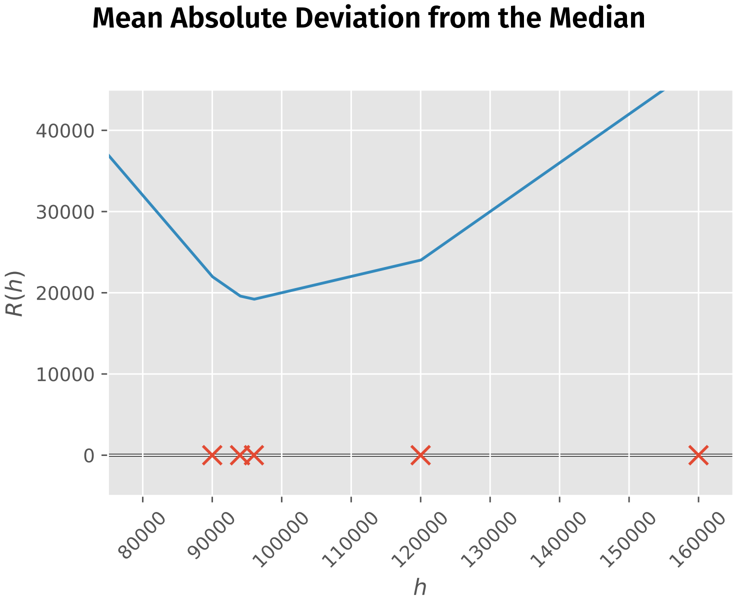
$$\frac{1}{n}\sum_{i=1}^{n}|y_i - \text{median}(y_1, y_2, \ldots, y_n)|$$

- It measures how far each data point is from the median, on average.

Question

For the data set $2, 3, 3, 4$, what is the mean absolute deviation from the median?

a) 0 b) $\frac{1}{2}$ c) 1 d) 2



Mean Absolute Deviation from the Median
R(h)
h
40000
30000
20000
10000
0
80000
90000
100000
110000
120000
130000
140000
150000
160000

Square Loss

- The empirical risk for the square loss is

$$R_{\mathrm{sq}}(h) = \frac{1}{n} \sum_{i=1}^{n} (y_i - h)^2.$$

- $R_{\mathrm{sq}}(h)$ is minimized at $h^* = \mathrm{mean}(y_1, y_2, \dots, y_n)$.

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- $R_{\text{sq}}(h)$ is minimized at $h^* = \text{mean}(y_1, y_2, \ldots, y_n)$.

- Therefore, the minimum value of $R_{\text{sq}}(h)$ is

$$\begin{aligned} R_{\text{sq}}(h^*) &= R_{\text{sq}}(\text{mean}(y_1, y_2, \ldots, y_n)) \\ &= \frac{1}{n}\sum_{i=1}^{n}(y_i - \text{mean}(y_1, y_2, \ldots, y_n))^2. \end{aligned}$$

Variance

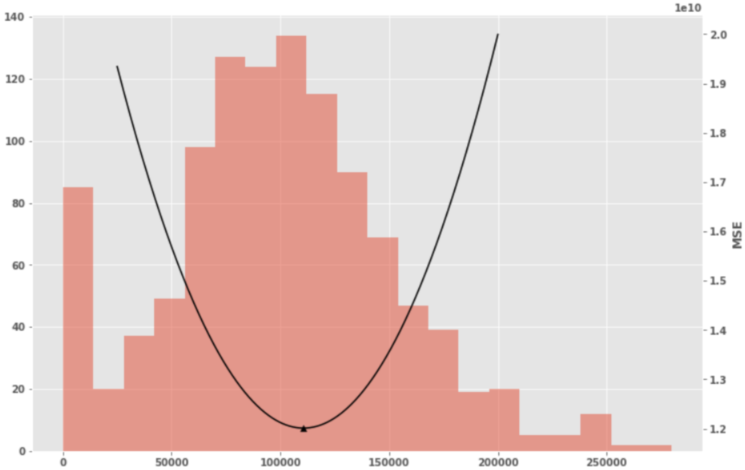
- The minimum value of $R_{\text{sq}}(h)$ is the mean squared deviation from the mean, more commonly known as the **variance**.

$$\frac{1}{n}\sum_{i=1}^{n}(y_i - \text{mean}(y_1, y_2, \ldots, y_n))^2$$

- It measures the squared distance of each data point from the mean, on average.

- Its square root is called the **standard deviation.**

Variance



1e10
MSE
140
120
100
80
60
40
20
0
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
0
50000
100000
150000
200000
250000

0-1 Loss

- The empirical risk for the 0-1 loss is

$$R_{0,1}(h) = \frac{1}{n} \sum_{i=1}^{n} \begin{cases} 0, & \text{if } h = y_i \\ 1, & \text{if } h \neq y_i \end{cases}$$

- This is simply a count of the number of data points not equal to h.
- $R_{0,1}(h)$ is minimized at $h^* = \text{mode}(y_1, y_2, \ldots, y_n)$.
- Therefore, $R_{0,1}(h^*)$ is a count of the number of data points not equal to the mode.

A Poor Way to Measure Spread

- The minimium value of $R_{0,1}(h)$ is the number of data points not equal to the mode.
- Higher value means less of the data is clustered at the mode.
- Just as the mode is a very simplistic way to measure the center of the data, this is a very crude way to measure spread.

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Question

For two different data sets, does it make sense say the data set with more data points not equal to the mode is more spread out?

Summary

- Different loss functions lead to empirical risk functions that are minimized at various measures of **center**.
- The minimum values of these risk runctions are various measures of **spread**.
- There are many different ways to measure both center and spread. These are sometimes called **descriptive statistics**.