DSC 40A

Theoretical Foundations of Data Science I

In This Video

Which prediction minimizes the mean error?

Recommended Reading

Course Notes: Chapter 1, Section 1

The Best Prediction

- We want the best prediction, h^*.
- Goal: find h that minimizes the mean error:

$$R(h) = \frac{1}{n} \sum_{i=1}^{n} |y_i - h|$$

- This is an optimization problem.

Question

Can we use calculus to minimize R?

Minimizing with Calculus

- Calculus: take derivative, set equal to zero, solve.

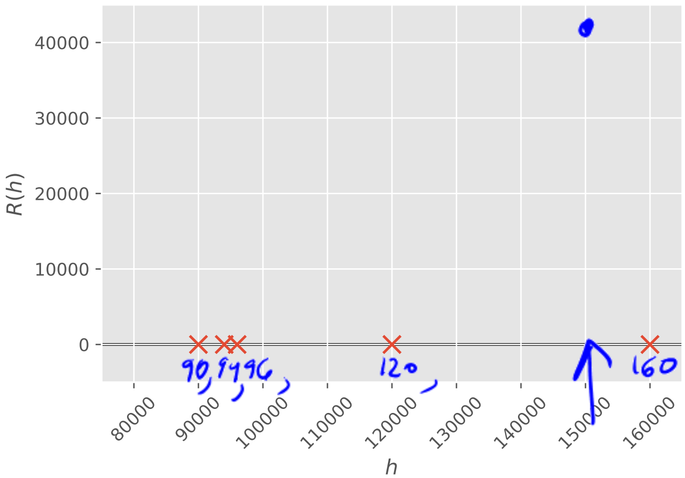
$$R(h) = \frac{1}{n} \sum_{i=1}^{n} |h - y_i|$$

$$R'(h) = \frac{1}{n} \sum_{i=1}^{n} \frac{d}{dh}\left(|h - y_i|\right)$$

problem: not differentiable because has cusp

Can't be minimized with calc.

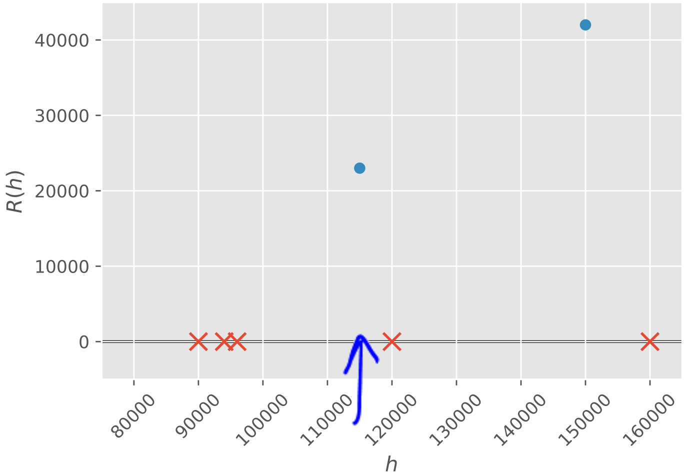
Plotting the Mean Error



Recall: $R(\underline{150{,}000}) = 42{,}000$ ← mean error

h

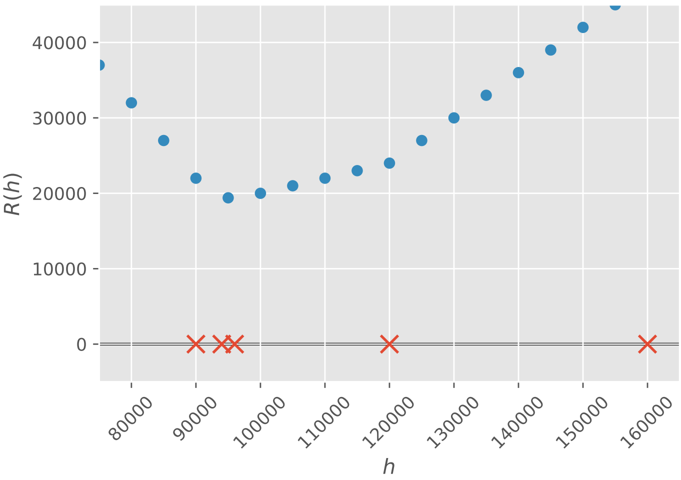
Plotting the Mean Error



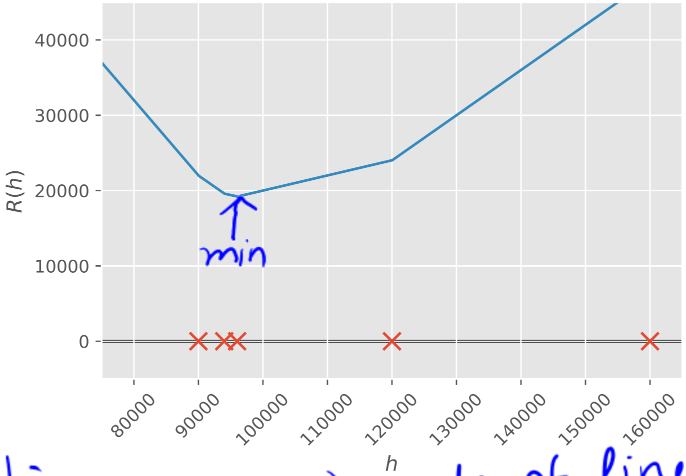
h $\$$

Recall: $R(115{,}000) = 23{,}000$

Plotting the Mean Error



Plotting the Mean Error



1) continuous

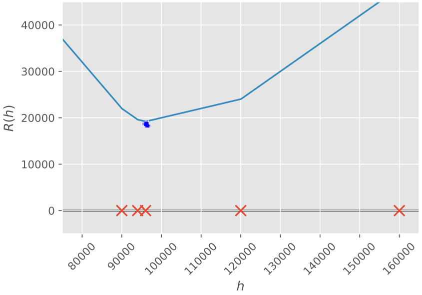
2) made of line segments (piecewise linear)

Question

A local minimum occurs when the slope of a function goes from ________________. Select all that apply.

A) positive to negative
B) negative to positive
C) positive to zero
D) negative to zero

Goal



- Find where slope of R goes from negative to non-negative.

pos or zero

- Want a formula for the slope of R at h.

or

Sums of Linear Functions

- Let

$$f_1(x) = 3x + 7 \qquad f_2(x) = 5x - 4 \qquad f_3(x) = -2x - 8$$

- What is the slope of $f(x) = f_1(x) + f_2(x) + f_3(x)$?

$$3x + 5x - 2x \quad + c$$

$$6x \quad + c$$

slope is 6

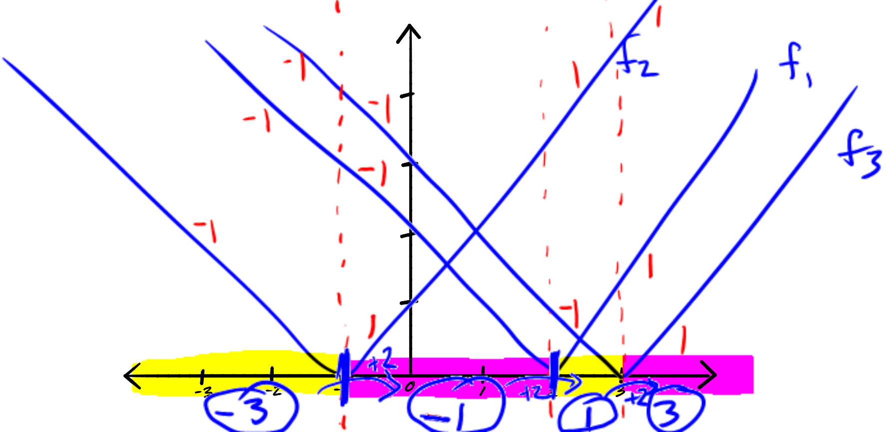
Sums of Absolute Values

- Let

$$f_1(x) = |x - 2| \qquad f_2(x) = |x + 1| \qquad f_3(x) = |x - 3|$$

2   -1   3

- What is the slope of $f(x) = f_1(x) + f_2(x) + f_3(x)$?



The Slope of the Mean Error

$R(h)$ is a sum of absolute value functions (times $\frac{1}{n}$):

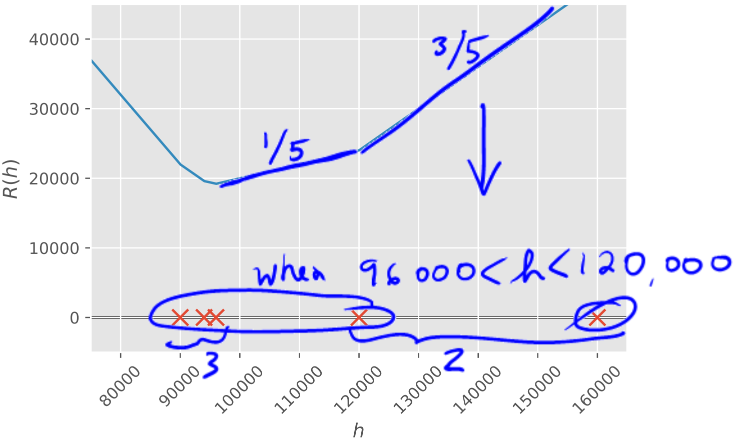
$$R(h) = \frac{1}{n}\left(|h - y_1| + |h - y_2| + \ldots + |h - y_n|\right)$$

$$\begin{aligned} R(h) &= \frac{1}{n}\sum_{i=1}^{n}|h-y_i| \\ &= \frac{1}{n}\left(\sum_{y_i<h}|\underbrace{h-y_i}_{\text{pos}}| + \sum_{y_i>h}|\underbrace{h-y_i}_{\text{neg}}| + \sum_{y_i=h}|\underbrace{h-y_i}_{0}|\right) \\ &= \frac{1}{n}\left(\underbrace{\sum_{y_i<h}(h-y_i)}_{\text{slope of each term is } 1} + \underbrace{\sum_{y_i>h}-(h-y_i)}_{\text{slope of each term is } -1}\right) \end{aligned}$$

The Slope of the Mean Error

The slope of R at h is:

$$\frac{1}{n}\left[\overbrace{(\#\text{ of } y_i\text{'s} < h)}^{\#\text{ terms in first sum}} - \overbrace{(\#\text{ of } y_i\text{'s} > h)}^{\#\text{ terms in 2nd sum}}\right]$$



Where the Slope's Sign Changes

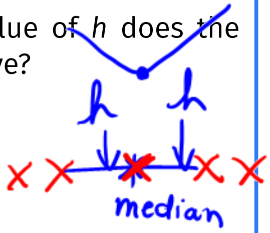
The slope of R at h is:

$$\frac{1}{n} \cdot \left[(\text{\# of } y_i\text{'s} < h) - (\text{\# of } y_i\text{'s} > h)\right]$$

Question

Suppose that n is odd. At what value of h does the slope of R go from negative to positive?

A) $h = \text{mean of } y_1, \dots, y_n$
B) $h = \text{median of } y_1, \dots, y_n$
C) $h = \text{mode of } y_1, \dots, y_n$



Summary: The Median Minimizes the Mean Error

- Our problem was: find h^* which minimizes the mean error, $R(h) = \frac{1}{n}\sum_{i=1}^{n} |y_i - h|$.

- The answer is: $\text{Median}(y_1, \ldots, y_n)$.

- The **best prediction**[1] is the **median**.

- **Next time:** We consider a different measure of error that is differentiable.

[1] in terms of mean error