DSC 40A - Homework 6

Due: Friday, February 25, 2022 at 11:59pm

Write your solutions to the following problems by either typing them up or handwriting them on another piece of paper. Homeworks are due to Gradescope by 11:59pm on the due date. You can use a slip day to extend the deadline by 24 hours.

Homework will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should **always explain and justify** your conclusions, using sound reasoning. Your goal should be to convince the reader of your assertions. If a question does not require explanation, it will be explicitly stated.

Homeworks should be written up and turned in by each student individually. You may talk to other students in the class about the problems and discuss solution strategies, but you should not share any written communication and you should not check answers with classmates. You can tell someone how to do a homework problem, but you cannot show them how to do it.

For each problem you submit, you should **cite your sources** by including a list of names of other students with whom you discussed the problem. Instructors do not need to be cited.

This homework will be graded out of 50 points. The point value of each problem or sub-problem is indicated by the number of avocados shown.

Note: For Problem 5 part (c), code your answer in the supplementary Jupyter notebook (linked). You'll need to turn in your completed Python file to Gradescope separately from the rest of this homework, in a file called `hw6code.py`. This part will be autograded, so no explanation is needed.

Throughout this homework, please give your **answers in two forms**:

- unsimplified, in terms of factorials, exponents, the permutation formula $P(n, k)$, and the combination formula $C(n, k) = \binom{n}{k}$, and
- as a decimal approximation to at least 3 decimal places.

Problem 1. Baseball

a) Suppose that your baseball team has 12 players but you only have 9 matching team hats. How many ways are there for you to select who gets a hat and who doesn't?

b) Suppose that your baseball team has 12 players and you have 15 jerseys, numbered 1 through 15. You will give out one jersey to each player, and three will be leftover. How many ways are there for you to assign jerseys to players?

c) As before, your baseball team has 12 players and you have 15 jerseys, numbered 1 through 15. You will give out one jersey to each player, and three will be leftover. How many ways are there for you to assign jerseys to players if you must give out jersey number 1?

d) Suppose that at the batting range, where a machine pitches the ball, each time you swing the bat, your probability of hitting the ball is $3/4$. If you swing 9 times, what is the probability that you hit the ball exactly 7 times?

e) Suppose that when a pitcher is throwing the ball, your probability of hitting the ball depends on the kind of pitch. Your probability of hitting the ball is

- $\frac{1}{2}$ for a fastball,
- $\frac{1}{3}$ for a breaking ball, and
- $\frac{1}{4}$ for a changeup

Suppose that at practice, a pitcher throws you three fastballs, three breaking balls, and three changeups. What is the probability that you miss one breaking ball and one changeup, but hit all 7 other balls?

f) Suppose that at practice, a pitcher throws you three fastballs, three breaking balls, and three changeups, and your probability of hitting each is the same as in part (e). What is the probability that you hit the ball exactly 7 times?

Hint: Use cases. Part (e) addresses one of the possible cases.

Problem 2. Be Unique

To break a tie among a group of $n \geq 3$ people, you come up with the following tiebreaker:

- Everyone flips a coin.
- If one person's coin is different from all the others, that person wins, and the tie is broken!
- Otherwise, repeat the process.

a) What is the probability that the tie is broken after the first coin toss?

b) Find the limit as n approaches infinity of the probability you found, and explain intuitively why this makes sense.

c) Fix an integer $k \geq 1$. Find the probability that the tie is broken after exactly k coin tosses?

Problem 3. 5 Surprise Mini Brands

A toy called 5 Surprise Mini Brands is a plastic capsule containing five miniature replicas of branded household products, like Skippy Peanut Butter, Dove Bodywash, Kikkoman Soy Sauce, or Breyers Ice Cream. Each capsule is a surprise; you don't know which minis it will contain until you open it! We'll assume for this problem that there are 70 possible products, each of which is manufactured in equal quantities, so you're no more likely to get any one product than any other.

a) Suppose that each capsule contains five **different** randomly selected products. If you buy two capsules, what is the probability that you end up with exactly five distinct products?

b) Suppose that each capsule contains five **different** randomly selected products. If you buy two capsules, what is the probability that you end up with exactly ten distinct products?

c) Suppose that each capsule contains five randomly selected products, any of which **could be the same**. That is, each of the five products is equally likely to be any of the 70 possible products. If you buy two capsules, what is the probability that you end up with exactly ten distinct products?

d) Suppose that each capsule contains five randomly selected products, any of which **could be the same**. That is, each of the five products is equally likely to be any of the 70 possible products. If you buy two capsules, what is the probability that you end up with exactly nine distinct products?

Problem 4. Probability Theory

Let S be a sample space, and let A, E_1, E_2, E_3 be events in that sample space. Suppose that $E_1 \cap E_2$, $E_1 \cap E_3$, and $E_2 \cap E_3$ are all empty. Given the following probabilities, find $P(E_2|A)$:

$$\begin{array}{ll} P(E_1) = 1/6 & P(A|E_1) = 1/9 \\ P(E_2) = 1/3 & P(A|E_2) = 1/7 \\ P(E_3) = 1/2 & P(A|E_3) = 1/5 \end{array}$$

Problem 5. Woven Words

Suppose you are given two words with no letters in common. Define a **woven word** created from these to be a word containing all the letters from both words, such that both words' letters appear in order from left to right. For example, a woven word formed from *avocado* and *green* is *greavoceando*. Bolding the letters of *green* makes this a little easier to see:

g r e *a v o c* **e** *a* **n** *d o*.

a) What are all woven words that can be formed from *go* and *cat*? No explanation needed.

b) Let $W(n, m)$ be the number of woven words formed from input words w_1 of length n and w_2 of length m. Explain why $W(n, m)$ satisfies each of the following:

$$W(0, m) = 1 \tag{1}$$

$$W(n, 0) = 1 \tag{2}$$

$$W(n, m) = W(n-1, m) + W(n, m-1) \quad \text{for } n > 0 \text{ and } m > 0. \tag{3}$$

c) In the supplementary Jupyter notebook (linked), fill in the blanks (marked by ...) in the provided partially complete `woven_words` function. This function should take as input two strings that have no letters in common and return an array containing all the different woven words that can be created from the two inputs.

d) Suppose your two input words are of length m and n. How many woven words can be formed? Justify your answer.