DSC 40A - Group Work Session 7

due November 18, 2021 at 11:59pm

Write your solutions to the following problems by either typing them up or handwriting them on another piece of paper. You must work in a group of 2 to 4 students for at least 50 minutes to get credit for this assignment. It's best to join a discussion section if possible.

One person from each group should submit your solutions to Gradescope by 11:59pm on Thursday. Make sure to **tag all group members** so everyone gets credit. This worksheet won't be graded on correctness, but rather on good-faith effort. Even if you don't solve any of the problems, you should include some explanation of what you thought about and discussed, so that you can get credit for spending time on the assignment.

Problem 1.

There are two boxes. Box 1 contains three red and five white balls and box 2 contains two red and five white balls. A box is chosen at random, with each box equally likely to be chosen. Then, a ball is chosen at random from this box, with each ball equally likely to be chosen. The ball turns out to be red. What is the probability that it came from box 1?

Problem 2.

Recall that two events E, F are **independent** if $P(E \cap F) = P(E)P(F)$. If $P(E \cap F) \neq P(E)P(F)$, then E and F are **dependent**.

a) Let A and B be mutually exclusive events in a sample space S. Suppose $0 < P(A) < 1$ and $0 < P(B) < 1$. Is it possible that A and B are independent? If yes, give an example, otherwise prove why not.

b) Let A and B be events in a sample space with $0 < P(A) < 1$ and $0 < P(B) < 1$. If A is a subset of B, can A and B be independent? If yes, give an example, otherwise prove why not.

Hint: If A is a subset of B, what is $P(B|A)$?

c) Consider two flips of a fair coin. The sample space, S, is $S =$ all outcomes of 2 flips of a coin $= \{HH, HT, TH, TT\}$. We define the event A as $A =$ first flip is heads $= \{HH, HT\}$, and the event B as $B =$ second flip is heads $= \{HH, TH\}$. You can verify that A and B are independent.

Now, suppose that the coin is not fair, and instead flips heads the first time with probability p and flips heads the second time with probability q.

Are A and B still independent? Prove your answer by showing that $P(A \cap B) = P(A)P(B)$ or that $P(A \cap B) \neq P(A)P(B)$.

Problem 3.

A box contains two coins: a regular coin and one fake two-headed coin ($P(H) = 1$). Choose a coin at random and flip it twice. Define the following events.

- A: First flip is heads (H).
- B: Second flip is heads (H).
- C: Coin 1 (regular) has been selected.

a) Are A and B independent?

Hint: Use the law of total probability. Note that this is a different problem than Problem 2c.

b) Two events E, F are **conditionally independent** given G if $P(E \cap F|G) = P(E|G)P(F|G)$.

Are A and B conditionally independent given C? Explain your answer intuitively.