
DSC 40A - Extra Practice Session 3
Wednesday, February 2, 2022

Problem 1. Matrix, Vector, Scalar, or Nonsense?

Suppose M is an $m \times n$ matrix, v is a vector in \mathbb{R}^n , and s is a scalar. Determine whether each of the following quantities is a matrix, vector, scalar, or nonsense.

a) Mv

b) vM

c) v^2

d) $M^T M$

e) MM^T

f) $v^T Mv$

g) $(sMv) \cdot (sMv)$

h) $(sv^T M^T)^T$

i) $v^T M^T M v$

j) $vv^T + M^T M$

Problem 2. Orthogonality

Two vectors are **orthogonal** if their dot product is 0, i.e. for $\vec{a}, \vec{b} \in \mathbb{R}^n$:

$$\vec{a}^T \vec{b} = 0 \implies \vec{a}, \vec{b} \text{ are orthogonal}$$

Orthogonality is a generalization of perpendicularity to multiple dimensions. (Two orthogonal vectors in 2D meet at a right angle.)

- a) Is it possible for a vector to be orthogonal to itself?

b) Show that if \vec{u} is orthogonal to both \vec{v} and \vec{w} , then \vec{u} is also orthogonal to any linear combination of \vec{v} and \vec{w} , $\alpha\vec{v} + \beta\vec{w}$.

c) Show that if $A^T\vec{b} = 0$, then \vec{b} is orthogonal to the **column space** of A , which is the space of all linear combinations of the columns of A .

Problem 3. Farmfluencer

Billy the avocado farmer heard about the success of 72 year-old Gerald Stratford's viral gardening videos on Twitter and Instagram. After witnessing Gerald turn into the so-called [King of Big Veg](#) overnight, Billy is feeling inspired to up his social media game (he's also feeling a little bit jealous).

Billy is new to Instagram and is trying to understand how people gain followers. In particular, he wants to be able to predict the number of followers, y , based on these features:

- number of people they follow, $x^{(1)}$
 - number of years since first post, $x^{(2)}$
 - average number of posts per day, $x^{(3)}$
- a) Suppose Billy has access to a large data set of Instagram accounts, and he uses multiple regression on this data to fit a linear prediction rule of the form

$$H(\vec{x}) = w_0 + w_1x^{(1)} + w_2x^{(2)} + w_3x^{(3)}.$$

What does w_2 represent in terms of Instagram followers?

- b) What if instead of the number of years since the first post, $x^{(2)}$, Billy instead uses the number of days since the first post, $x^{(4)}$. Now he uses multiple regression to fit a prediction rule of the form

$$H'(\vec{x}) = w'_0 + w'_1x^{(1)} + w'_3x^{(3)} + w'_4x^{(4)}.$$

How do the parameters of this prediction rule (w'_0, w'_1, w'_3, w'_4) compare to the parameters of original prediction rule (w_0, w_1, w_2, w_3) ?