DSC 40A

Theoretical Foundations of Data Science I

Last Time

- To predict future salary:
  - Gather salaries $y_1, y_2, \ldots, y_n$.
  - Find a prediction h^* which minimizes the **mean error**:

$$R(h) = \frac{1}{n} \sum_{i=1}^{n} |y_i - h|$$

  - We saw that $R(h)$ is minimized by $\text{Median}(y_1, \ldots, y_n)$.
- We turned learning into a math problem and solved it.

Two things we don't like

1. **Minimizing** the mean error wasn't so easy.

2. Actually **computing** the median isn't so easy, either.

In This Video

Is there another way to measure the quality of a prediction that avoids these problems?

Recommended Reading

Course Notes: Chapter 1, Section 1

The mean error is not differentiable

- We can't compute $\frac{d}{dh}|y_i - h|$.

- Remember: $|y_i - h|$ measures how far h is from y_i.

- Is there something besides $|y_i - h|$ which:
  1. Measures how far h is from y_i, *and*
  2. is **differentiable**?

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Question

Which of these would work?

a) $e^{|y_i - h|}$

b) $|y_i - h|^2$

c) $|y_i - h|^3$

d) $\cos(y_i - h)$

The **Squared Error**

- Let h be a prediction and y be the right answer. The **squared error** is:

$$|y - h|^2 = (y - h)^2$$

- Like error, measures how far h is from y.

- But unlike error, the squared error is **differentiable**:

$$\frac{d}{dh}(y - h)^2 =$$

The Mean Squared Error

- Suppose we predicted a future salary of $h_1 = 150{,}000$ *before* collecting data.

salary	error of h_1	squared error of h_1
90,000	60,000	$(60{,}000)^2$
94,000	56,000	$(56{,}000)^2$
96,000	54,000	$(54{,}000)^2$
120,000	30,000	$(30{,}000)^2$
160,000	10,000	$(10{,}000)^2$

total squared error: 1.0652×10^{10}

mean squared error: 2.13×10^9

- A good prediction is one with small **mean squared error**.

The Mean Squared Error

- Now suppose we had predicted $h_2 = 115{,}000$.

salary	error of h_2	squared error of h_2
90,000	25,000	$(25{,}000)^2$
94,000	21,000	$(21{,}000)^2$
96,000	19,000	$(19{,}000)^2$
120,000	5,000	$(5{,}000)^2$
160,000	45,000	$(45{,}000)^2$

total squared error: 3.47×10^9
mean squared error: 6.95×10^8

- A good prediction is one with small **mean squared error**.

The New Idea

- Make prediction by minimizing the **mean squared error**:

$$R_{\text{sq}}(h) = \frac{1}{n}\sum_{i=1}^{n}(y_i - h)^2$$

- Strategy: Take derivative, set to zero, solve for minimizer.

The New Idea

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Question

Which of these is dR_{sq}/dh?

a) $\frac{1}{n}\sum_{i=1}^{n}(y_i - h)$

b) 0

c) $\sum_{i=1}^{n} y_i$

d) $\frac{2}{n}\sum_{i=1}^{n}(h - y_i)$

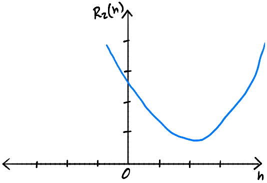
Solution

$$\frac{dR_{\text{sq}}}{dh} = \frac{d}{dh}\left[\frac{1}{n}\sum_{i=1}^{n}(y_i - h)^2\right]$$

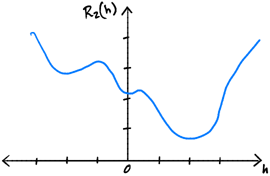
Set to zero and solve for minimizer

Question

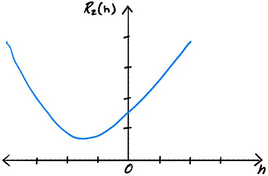
Suppose $y_1, \ldots, y_n$ are salaries. Which plot could be $R_{\text{sq}}(h)$?



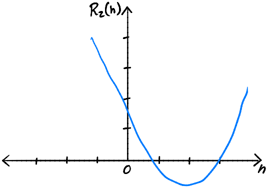
(a)



(b)



(c)



(d)

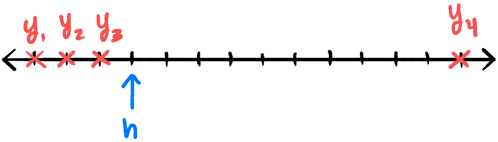
Bonus: the mean is easy to compute

```
def mean(numbers):
    total = 0
    for number in numbers:
        total = total + number
    return total / len(numbers)
```

- Time complexity: $\Theta(n)$
- Median by sorting: $\Theta(n \log n)$
- But there's a $\Theta(n)$ way to find median: `quickselect`.
- DSC 40B.

Outliers

- The mean is quite **sensitive** to outliers.

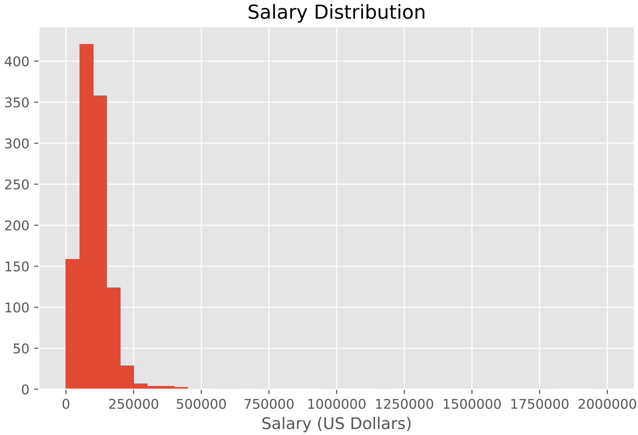


- $|y_4 - h|$ is 10 times as big as $|y_3 - h|$.
- But $(y_4 - h)^2$ is 100 times as big as $(y_3 - h)^2$.
- Squared error can be dominated by outliers.

Example: Data Science Salaries

- Data set of 1121 self-reported data science salaries in the United States from the 2018 StackOverflow survey.
- Median = \$100,000
- Mean = \$111,032
- Max = \$2,000,000
- Min = \$52
- 95th Percentile: \$200,000

Example: Data Science Salaries



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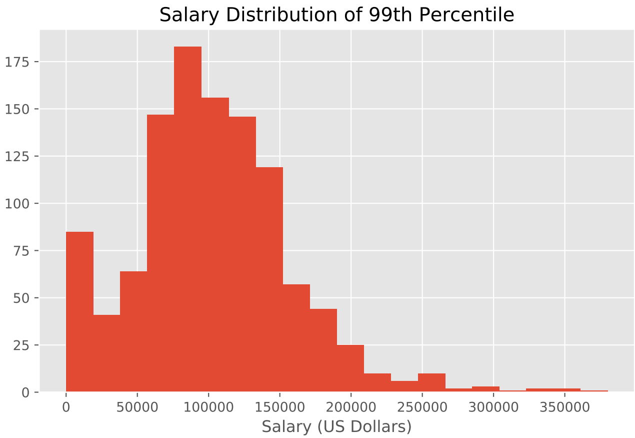


Example: Income Inequality

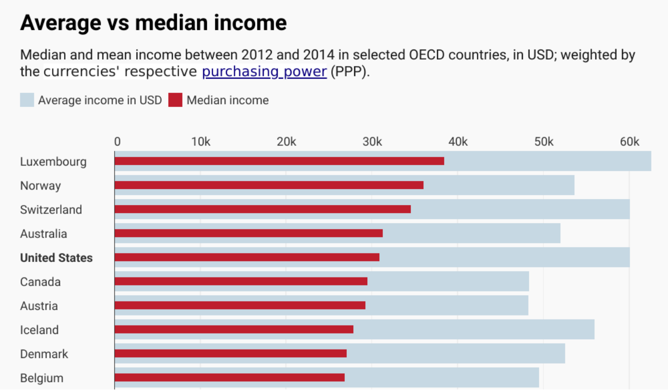
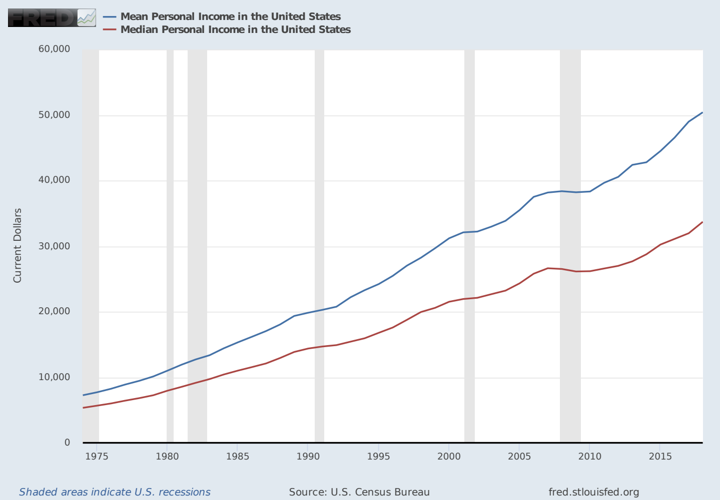


Chart: Lisa Charlotte Rost, Datawrapper

Example: Income Inequality



Summary: The Mean Minimizes the Mean Squared Error

- Our problem was: find h^* which minimizes the mean squared error, $R_{\text{sq}}(h) = \frac{1}{n}\sum_{i=1}^{n}(y_i - h)^2$.
- The answer is: $\text{Mean}(y_1, \ldots, y_n)$.
- Using mean squared error biases the prediction towards outliers.
- **Next time:** We consider both the mean error and the mean squared error as part of a more general framework.