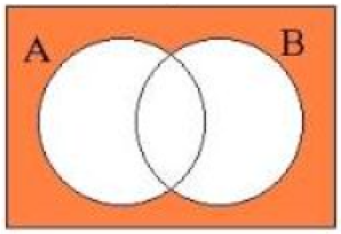
DSC 40A - Extra Practice Session 5
Wednesday, March 2, 2022

Problem 1. Complements of Independent Events are Independent

Let A and B be two independent events in the sample space S. Show that $\bar{A}$ and $\bar{B}$ must be independent of one another.

You may use the fact that $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$, which should be apparent from the Venn diagram below.

try it - 5 mins



algebraic proof

$$P(A \cup B) - P(A) + P(B) - P(A \cap B)$$

trying to prove $\underbrace{P(\bar{A} \cap \bar{B})}_{\text{start here}} = \underbrace{P(\bar{A}) * P(\bar{B})}_{\text{hopefully end here}}$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) \rightarrow \text{given (hint)}$$

$$= 1 - \left(P(A) + P(B) - P(A \cap B)\right) \quad \text{add. rule}$$

$$= 1 - \left(P(A) + P(B) - P(A) * P(B)\right) \quad \text{distribute}$$

$$= 1 - P(A) - P(B) + P(A) * P(B)$$

? $(\vdots)$ $= 1 - P(A) - P(B) + P(A) * P(B)$

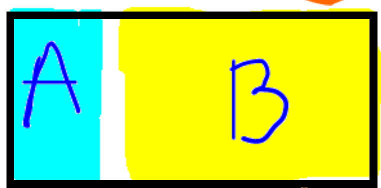
$\therefore \neq (1 - P(A))(1 - P(B)) = P(\bar{A}) * P(\bar{B})$ factor

Problem 2. Visualizing Independence and Conditional Independence

Let's represent a sample space S as a rectangle with area one. Then we'll represent events within that sample space as regions with area equal to their probability.

a) For the sample space S shown below, draw two **mutually exclusive** events A and B with $P(A) = \frac{1}{4}$ and $P(B) = \frac{2}{3}$.

disjoint / no overlap



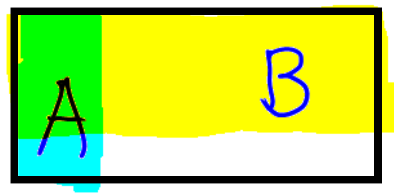
are these independent? $P(A \cap B) \overset{?}{=} P(A) * P(B)$, with $P(A \cap B) = 0$, $P(A) = 1/4$, $P(B) = 2/3$

NO: showed in groupwork

b) For the sample space S shown below, draw two **independent** events A and B with $P(A) = \frac{1}{4}$ and $P(B) = \frac{2}{3}$.

def. of ind:

- $P(A \cap B) = P(A) * P(B)$
- $P(A|B) = P(A)$, $\frac{1}{4} = \frac{1}{4}$ ←
- $P(B|A) = P(B)$, $\frac{2}{3} = \frac{2}{3}$ ←



$P(A|B) = 1/4$ means A takes up a quarter of B's area

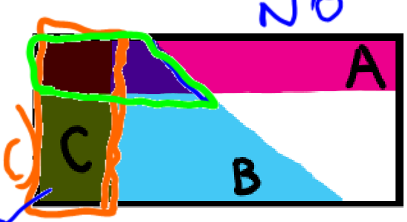
c) In the sample space S shown below, are A and B independent? Are A and B conditionally independent given C?

YES NO

conditional ind:

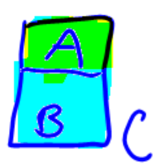
$P((A \cap B)|C) = P(A|C) * P(B|C)$, with $1/3$, $1/3$

within C, are A and B independent?



$P(A) = 1/3$

$P(A|B) = 1/4 < 1/3$

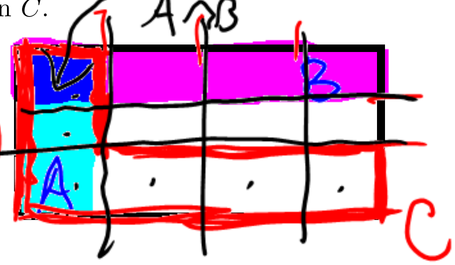


d) For the sample space S shown below, draw events A, B, and C such that A and B are independent but conditionally dependent given C.

portion of B taken up by A

ind: $\underbrace{P(A \cap B)}_{1/12} = \underbrace{P(A)}_{1/4} * \underbrace{P(B)}_{1/3}$

$1/12 = 1/4 * 1/3$



cond dep given C:

$\underbrace{P((A \cap B)|C)}_{1/6} \neq \underbrace{P(A|C)}_{1/2} * \underbrace{P(B|C)}_{1/6}$

$1/6 \neq 1/2 * 1/6$

Problem 3.

In parts of the world other than San Diego, the weather changes from day to day. In these places, people try to guess tomorrow's weather using the current conditions.

Weather data for 20 random days in Columbus, Ohio are recorded below, along with the next day's weather (rainy, cloudy, or sunny).

Suppose that today's humidity is > 50%, the temperature is hot, and the air pressure is low. Use Naive Bayes (without smoothing) to predict whether tomorrow will be rainy, cloudy, or sunny. Show your work.

class | features

Next Day's Weather	Humidity	Temperature	Air Pressure
Rainy	> 50%	Cool	Low
Rainy	> 50%	Hot	Low
Rainy	> 50%	Cool	Low
Rainy	25%-50%	Hot	High
Rainy	25%-50%	Hot	Low
Rainy	25%-50%	Cool	Low
Rainy	25%-50%	Cool	Low
Rainy	< 25%	Cool	Low
Rainy	< 25%	Hot	Low
Rainy	< 25%	Hot	High
Cloudy	> 50%	Cool	Low
Cloudy	> 50%	Cool	Low
Cloudy	25%-50%	Hot	High
Cloudy	< 25%	Cool	High
Cloudy	< 25%	Cool	Low
Sunny	> 50%	Cool	Low
Sunny	> 50%	Hot	High
Sunny	> 50%	Cool	High
Sunny	25%-50%	Hot	High
Sunny	< 25%	Hot	High

Bayes

$$P(B|A) = \frac{P(A|B) * P(B)}{P(A)}$$

$$P(\text{class} \mid \text{features}) = \frac{P(\text{features} \mid \text{class}) * P(\text{class})}{P(\text{features})}$$

for each class (rainy, cloudy, sunny)

$$P(\text{rainy} \mid >50, \text{hot}, \text{low}) \propto P(>50, \text{hot}, \text{low} \mid \text{rainy}) * P(\text{rainy})$$

proportional to

$$= \underbrace{P(>50 \mid \text{rainy})}_{3/10} * \underbrace{P(\text{hot} \mid \text{rainy})}_{5/10} * \underbrace{P(\text{low} \mid \text{rainy})}_{8/10} * P(\text{rainy})$$

$* \frac{10}{20}$

same for cloudy, sunny

rainy wins!