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DSC 40A - Extra Practice Session 4

Wednesday, February 16, 2022

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**Problem 1. UCSD Phone Numbers**

All UCSD campus phone numbers take the form 858-534-XXXX.

a) What is the probability of a randomly chosen UCSD phone number including the number 7?

try it

opposite: no 7's

$$1 - P(\text{no 7s})$$

$$\frac{9}{10} * \frac{9}{10} * \frac{9}{10} * \frac{9}{10}$$

$$1 - \left(\frac{9}{10}\right)^4$$

b) What is the probability of a randomly chosen UCSD phone number containing no 0s, 1s, or 2s?

try it

opposite: there is a 0, 1, or 2

restatement:

all 3, 4, 5, 6, 7, 8, 9

7 numbers

$$\left(\frac{7}{10}\right)^4$$

AND

which rule?

c) What is the probability of a randomly chosen UCSD phone number containing exactly four 7s if we know that it contains at least three 7s??

try it

F

E

Addition Rule:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Multiplication Rule:

$$P(A \cap B) = P(A) \cdot P(B|A)$$

Complement Rule:

$$P(\bar{A}) = 1 - P(A)$$

Conditional Probability:

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

numerator:  $P(E \cap F) = P(E)$   
 $= \left(\frac{1}{10}\right)^4$

denominator:  $P(\text{at least 3 7s})$   
5 cases: (all disjoint)  
1. 7777  $\rightarrow \left(\frac{1}{10}\right)^4$   
2. 777Y  $\rightarrow \left(\frac{1}{10}\right)^3 \cdot \frac{9}{10} = \frac{9}{10^4}$   
3. 77Y7  
4. 7Y77  
5. Y777  
total  $4 \cdot \frac{9}{10^4} + \frac{1}{10^4} = \frac{37}{10^4}$

d) What is the probability of a randomly chosen UCSD phone number having the last four digits all distinct?

another way to do (c):

$$\frac{\# \text{ phone numbers with 4 7's}}{\# \text{ phone numbers with } \geq 3 \text{ 7's}} = \frac{1}{37}$$

$$\frac{\frac{1}{37}}{\frac{1}{10^4}} = \frac{1}{37/10^4}$$

e) What is the probability of a randomly chosen UCSD phone number having the last seven digits all distinct?

try it 858-(534-XXXX)

Xs could be 0, 1, 2, 6, 7, 8, 9

$$\frac{7}{10} * \frac{6}{10} * \frac{5}{10} * \frac{4}{10} \leftarrow \text{multiplication rule}$$

f) What is the probability of two randomly chosen UCSD phone numbers having the same last digit?

$10 * \frac{1}{100}$  or  $\frac{1}{10}$   
 cases both end in 0, 1, 2, ..., 9  
 picking 2 last digits  
 picking 1 last digit

g) What is the probability of two randomly chosen UCSD phone numbers having the same last digit or the same second-to-last digit?

try it - pm answer

careful of overlap

$\frac{1}{10} + \frac{1}{10}$  but the cases overlap  
 $\frac{1}{10} + \frac{1}{10} - \left(\frac{1}{10}\right)^2$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

**Problem 2. Habla Espanol?**

In your Spanish conversation class, the instructor randomly selects students to answer questions. You're covering in the back of the room, hoping you never get called on.

- a) If there are 25 students in your class and your instructor asks 6 questions, what is the chance that you are called on? Assume that for each question, any student is equally likely to be chosen, regardless of whether they have already answered another question.

try it - pm answer

many cases

Complement: never called on each of the six times, it's someone else

$$1 - \left(\frac{24}{25}\right)^6$$

- b) If there are 25 students in your class and your instructor asks 6 questions, what is the chance that you are called on? Assume that for each question, any student who has not yet been called on is equally likely to be chosen. Students who have been called on cannot be called on again.

try it

$$1 - P(\text{you're never called on})$$

$$1 - \left(\frac{24}{25}\right) * \left(\frac{23}{24}\right) * \left(\frac{22}{23}\right) *$$

$$\left(\frac{21}{22}\right) * \left(\frac{20}{21}\right) * \left(\frac{19}{20}\right)$$

$$= 1 - \frac{19}{25} = \boxed{\frac{6}{25}} \leftarrow$$