DSC 40A - Extra Practice Session 4
Wednesday, February 16, 2022

Problem 1. UCSD Phone Numbers

All UCSD campus phone numbers take the form 858-534-XXXX.

a) What is the probability of a randomly chosen UCSD phone number including the number 7?

complement $1 - P(\text{no 7s})$

$1 - \left(\frac{9}{10}\right)^4$

1 7
or 2 7s
or 3 7s
or 4 7s

$\left(\frac{1}{10}\right)^4 = \frac{1}{10} * \frac{1}{10} * \frac{1}{10} * \frac{1}{10} \leftarrow P(\text{all 7s})$

b) What is the probability of a randomly chosen UCSD phone number containing no 0s, 1s, or 2s?

Complement?

all 3,4,5,6,7,8,9

$\left(\frac{7}{10}\right)^4$

mult. rule

$\frac{7}{10} * \frac{7}{10} * \frac{7}{10} * \frac{7}{10}$

c) What is the probability of a randomly chosen UCSD phone number containing exactly four 7s if we know that it contains at least three 7s??

E (exactly four 7s), F (at least three 7s)

Addition Rule:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Multiplication Rule:

$$P(A \cap B) = P(A) \cdot P(B|A)$$

Complement Rule:

$$P(\overline{A}) = 1 - P(A)$$

Conditional Probability:

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{P(E)}{P(F)}, \quad P(E) \to \left(\frac{1}{10}\right)^4$$

(= in this problem)

denominator: P(at least 3 7s)

cases:

three 7s:
- 777Y
- 77Y7
- 7Y77
- Y777

→ $\left(\frac{1}{10}\right)^3 \cdot \left(\frac{9}{10}\right)$, Y ≠ 7

or four 7s:
- 7777 → $\left(\frac{1}{10}\right)^4$

$$4\left(\frac{1}{10}\right)^3 \cdot \frac{9}{10} + \left(\frac{1}{10}\right)^4 = 37/10^4$$

$$P(E|F) = \frac{1/10^4}{37/10^4} = \boxed{1/37}$$

trick/shortcut: since all equally likely:

$$\frac{\#\text{ with 4 7's}}{\#\text{ with} \geq 3 \text{ 7's}} = \boxed{\frac{1}{37}}$$

direct interpretation

d) What is the probability of a randomly chosen UCSD phone number having the last four digits all distinct?

858-534-XXXX

$$\frac{10}{10} * \frac{9}{10} \times \frac{8}{10} * \frac{7}{10}$$

858-534-XXXX

A B C D

e) What is the probability of a randomly chosen UCSD phone number having the last seven digits all distinct?

$\frac{7}{10} * \frac{6}{10} * \frac{5}{10} * \frac{4}{10}$

$P(A \cap B) = P(A) * P(B|A)$

f) What is the probability of two randomly chosen UCSD phone numbers having the same last digit?

$\frac{1}{100}$ vs. $\frac{1}{10}$ ← trick! think of 1st phone number as fixed

$P(\text{1st phone \# ends in 7 and 2nd phone \# ends in 7}) = \frac{1}{10} * \frac{1}{10} * 10$ ← choose ending digit

g) What is the probability of two randomly chosen UCSD phone numbers having the same last digit or the same second-to-last digit?

B A

$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{10} + \frac{1}{10} - \frac{1}{100}$

$= 19/100$

Problem 2. Habla Espanol?

In your Spanish conversation class, the instructor randomly selects students to answer questions. You're cowering in the back of the room, hoping you never get called on.

a) If there are 25 students in your class and your instructor asks 6 questions, what is the chance that you are called on? Assume that for each question, any student is equally likely to be chosen, regardless of whether they have already answered another question.

(with replacement)

try it

complement: you're never called on

$$1 - \left(\frac{24}{25}\right)^6 \approx 0.21$$

b) If there are 25 students in your class and your instructor asks 6 questions, what is the chance that you are called on? Assume that for each question, any student who has not yet been called on is equally likely to be chosen. Students who have been called on cannot be called on again.

$$1 - \left(\frac{24}{25} * \frac{23}{24} * \frac{22}{23} * \frac{21}{22} * \frac{20}{21} * \frac{19}{20}\right)$$

$$1 - \frac{19}{25} = \boxed{\frac{6}{25}}$$

← of the 25 students, 6 will get chosen

$$\approx 0.24$$